ALGEBRA: LOGS & SURDS (28th January 2013)

Lesson Description

In this lesson, we will

- Revision the Exponential Laws
- Simplify surds
- Define logarithms
- Use the laws of logarithms to solve problems

Key Concepts

Exponential Laws

In previous grades we used the exponential laws to simplify expressions and solve exponential equations. Make sure you are familiar with these laws and definitions.

	Law	Example
1	$a^x.a^y = a^{x+y}$	$m^3.m^5 = m^8$
2	$\frac{a^x}{a^y} = a^{x-y}$	$\frac{p^{12}}{p^7} = p^5$
3	$(a^x)^y = a^{xy}$	$(n^3)^5 = n^{15}$
4	$(a.b)^x = a^x.b^x$	$(t.w)^{-2} = t^{-2}.w^{-2}$
5	$(\frac{a}{b})^x = \frac{a^x}{b^x}$	$(\frac{p}{3})^3 = \frac{p^3}{27}$

Here are some other useful definitions:

- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$
- $1^x = 1.$

(Maths for All, Chapter 6, Page 135)

Surds

A surd is a number of the form $a + b\sqrt[n]{c}; c \geq 0$, where $\sqrt[n]{c}$ is irrational.
Examples of surds are: $3 + \sqrt{2}$; $5 - \sqrt{7}$; $\sqrt{3}$; $\sqrt[3]{4}$; $4 - \sqrt[4]{9}$; $2 + 3\sqrt{10}$
Note: $a^{\frac{1}{n}} . b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$

	Laws	Examples
1.	$\sqrt[n]{a} . \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{10}$
2.	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	$\frac{\sqrt[5]{64}}{\sqrt[5]{2}} = \sqrt[5]{\frac{64}{2}} = \sqrt[5]{32} = 2$
3.	$\sqrt[p]{\sqrt[q]{a}} = \sqrt[pq]{a}$	$\sqrt[2]{\sqrt[3]{7}} = \sqrt[2\times 3]{7} = \sqrt[6]{7}$
4.	$(\sqrt[p]{a})^q = \sqrt[p]{a^q}$	$(\sqrt[3]{8})^2 = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Definitions (for n a natural number, $n \geq 2$)					
1.	$a^{\frac{1}{n}} = \sqrt[n]{a}$	2.	$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}$	3.	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(Clever Keeping Maths Simple. Grade 11 Learner's Book Page 15)

What is a logarithm?

The logarithm (to the base a) of a number x is the exponent y to which the base a must be raised to equal x. There are the following restrictions on a and x: a > 0, a ≠ 1 and x> 0. So if $\log_a x = y$ then $a^y = x$.

Laws of logarithms

- log (x.y) = log x + log y
- $\log\left(x \div y\right)$ = log x - log y
- $\log x^m = m.\log x$

Questions

Question 1

Simplify the following exponential expression by applying the exponential laws:

a.) $\frac{3^n+3^{n+1}}{7.3^n}$

b.) $\frac{5^{2n+1}.3^{2n-3}}{15^{2n}}$

Question 2

Simplify the following:

a.) $\sqrt{75}$

b.) $\sqrt[6]{64}$

c.) $\frac{8-\sqrt{80}}{12}$

Question 3

Find the value of y in the examples below:

a.) $y = \log_2 16$

b.) $y = \log_2 2$

c.) $y = \log_2 \frac{1}{8}$

d.) $y = \log_{16} 4$

Question 4

Solve the following equations

a.) $\log_7 3x = \log_7 (x + 4)$

b.) $\log_2 5x = \log_2 (x^2 - 36)$

Question 5

Express each of the following expressions as single logs and simplify, if possible.

a.) $\log_3 9 - \log_3 27$

b.) $7 \log m - 2 \log m + 3 \log 2m$