REVISION: ALGEBRA – SURDS, LOGS, SEQUENCES AND SERIES

11 MARCH 2013

Lesson Description

In this lesson, we revise how to:

- Simplify surds
- Use the laws of logarithms to solve equations
- Identify arithmetic, quadratic and geometric sequences
- Use sigma notation for series
- Find when a geometric series converges

Key Concepts

Surds

A surd is a number of the form $a + b\sqrt[n]{c}$; $c \geq 0$, where $\sqrt[n]{c}$ is irrational.

Examples of surds are: $3+\sqrt{2}$; $5-\sqrt{7}$; $\sqrt{3}$; $\sqrt[3]{4}$; $4-\sqrt[4]{9}$; $2+3\sqrt{10}$

Note: $a^{\frac{1}{n}} . b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$

	Laws	Examples
1.	$\sqrt[n]{a} . \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{10}$
2.	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	$\frac{\sqrt[5]{64}}{\sqrt[5]{2}} = \sqrt[5]{\frac{64}{2}} = \sqrt[5]{32} = 2$
3.	$\sqrt[p]{\sqrt[q]{a}} = \sqrt[pq]{a}$	$\sqrt[2]{\sqrt[3]{7}} = \sqrt[2\times 3]{7} = \sqrt[6]{7}$
4.	$(\sqrt[p]{a})^q = \sqrt[p]{a^q}$	$(\sqrt[3]{8})^2 = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Definitions (for n a natural number, $n \geq 2$)					
1.	$a^{\frac{1}{n}} = \sqrt[n]{a}$	2.	$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}$	3.	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(Clever Keeping Maths Simple. Grade 11 Learner's Book Page 15)

What is a logarithm?

The logarithm (to the base a) of a number x is the exponent y to which the base a must

be raised to equal x. There are the following restrictions on a and x: a > 0, a ≠ 1 and x> 0.

So if $\log_a x = y$ then $a^y = x$.

Laws of logarithms

- log (x.y) = log x + log y
- log $\left(x \div y \right)$ = log x - log y
- log x^m = m.log x

Question 2

Simplify the following:

a.) $\sqrt{75}$

b.) $\sqrt[6]{64}$

c.) $\frac{8-\sqrt{80}}{12}$

Question 5

Express each of the following expressions as single logs and simplify, if possible.

a.) $\log_3 9 - \log_3 27$

b.) 7 log m - 2 log m + 3 log 2m

Arithmetic Number Patterns

- $T_2 - T_1 = T_3 - T_2$
- $There\ is\ a\ constant\ difference\ between\ terms$
- $T_n = a + (n-1)d$
- $S_n = \frac{n}{2}(2a + (n-1)d$

Geometric Number Patterns

- $\frac{T_3}{T_2} = \frac{T_2}{T_1}$
- $There\ is\ a\ constant\ ratio\ between\ terms$
- $T_n = ar^{n-1}$
- $S_n = \frac{a(r^n-1)}{r-1}$

Convergent Geometric Number Patterns

- $-1 < r < 1$
- $S_\infty = \frac{a}{1-r}$

Question 1

Given the sequence $26; 24; 22; ...; -12$

a.) Determine the number of terms in the sequence. (3)

b.) How many terms must be added to get a sum of –1224? (5)

Question 3

3.1 Calculate n if: $\sum_{k=1}^{n}(3k-5) = 1715$ (5)

3.2 For which values of k will the series:

$(2k+1) + (2k+1)^2 + (2k+1)^3$ Converge? (3)