REVISION: FINANCIAL MATHEMATICS & TRIGONOMETRY

18 MARCH 2013

Lesson Description

In this lesson we revise:

- Various financial mathematics calculations including compound interest, investments and loan repayments.
- Compound and double angle trigonometric identities

Key Concepts

Compound Interest

Interest at the end of each term is added to the total and so the principal amount increases each term and the next amount of interest increases too.

Make sure that the percentage interest and the period of compounding are aligned. E.g. Interest that is quoted per annum (every year) must be divided by 12 if the interest is compounded monthly.

Formula for Compound Interest

$$A = P(1 + r)^n$$

A – accumulated amount P – principal r – rate as a % n – number of periods

Questions

Question 1

A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000.

Question 2

A lump sum of money is invested at 11.72% p.a. compounded quarterly. How long should it be invested in order for it to double?

Key Concepts

Example B

Definition: Annuity

The term annuity is used in financial mathematics to refer to any terminating sequence of REGULAR fixed payments over a SPECIFIED period of time.

- Ordinary Annuity: This is an annuity whose payments are made at the END of each period.
- (end of the week, month, half year, year, etc)
- Annuity Due: This is an annuity whose payments are made at the BEGINNING of each period.

Formula

Future Value Annuity: Investment

$$F_v = x\left[\frac{(1+i)^n - 1}{i}\right]$$

Example A

- A **sinking fund** is just an example of a future value annuity in which we invest cash so that we have a particular amount of money available when we need it. For example, managers in a business know that they will need to replace a certain piece of machinery in 5 years time. So what do they do?
- Machinery is purchased at a cost of R250 000 and is expected to rise in cost at 15%p.a.compound interest, and depreciate in value at a rate of 8% p.a. compounded annually.
- A sinking fund is started to make provision for replacing the old machine. The sinking fund pays 16% p.a. compounded monthly, and you make payments into this ordinary annuity for 5 years.

Example B

Loan Repayments

- Often, when we take a loan we have to pay back a certain amount each month until the loan is paid off.
- The amount we pay back each month is also called an annuity. It is called a PRESENT VALUE annuity.
- When the principal and the interest are paid by means of an annuity, we say that the debt is **amortised**.

Formula: Present Value Annuity: Loan Repayment

$$P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Questions

Question 1

(Adapted from Grade 12 Department of Education Nov 2009)

A car that costs R130 000 is advertised in the following way: “No deposit necessary and first payment due three months after date of purchase”

The interest rate quoted is 18% p.a. compounded monthly.

a.) Calculate the amount owing two months after the purchase date.

b.) John bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.
   i.) Calculate his monthly repayments
   ii.) Calculate the total of all John’s payments

c.) Paul also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car.
   i.) Calculate the total of all Paul’s repayments.
   ii.) Calculate the difference between John and Paul’s total repayments.

Key Concepts

Compound Angle Formulae

- $\sin(A + B) = \sin A\cos B + \cos A\sin B$
- $\sin(A - B) = \sin A\cos B - \cos A\sin B$
- $\cos(A + B) = \cos A\cos B - \sin A\sin B$
- $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Double Angle Formulae

- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
- $\sin 2A = 2\sin A\cos A$

Questions

Question 4

If sin 12° = k, then determine the following in terms of k:

a.) sin 192°

b.) cos 258°

c.) sin 336°