# FUNCTIONS & INVERSES 29 APRIL 2013

## Lesson Description

In this lesson, we:

- Investigate when the inverse of a function is a function and when it is a relation
- Solve exam type question on functions

# Key Concepts

## Terminology

- A function is a mathematical rule that maps an input value to a unique output value.
- The domain of a function is the set of all input values
- The range of a function is the set of all output values

## Vertical Line Test

We use a ruler to perform the "vertical line test" on a graph to see whether it is a function or not. Hold a clear plastic ruler parallel to the $y$-axis, i.e. vertical.

Move it from left to right over the axes.

If the ruler only ever cuts the curve in one place only throughout the movement from left to right, then the graph is a function.

If the ruler ever passes through two or more points on the graph, the graph will not be a function but a relation.

## Inverse of a function

An inverse of a function is a mapping of all the output values to the input values. The inverse of a function may not be a function. It is a reflection about the line y = x

# Questions

## Question 1

Consider the function $f(x) = \frac{3}{x-1} - 2$

a.) Write down the equations of the asymptotes of *f*.

b.) Calculate the intercepts of the graph of *f* with the axes.

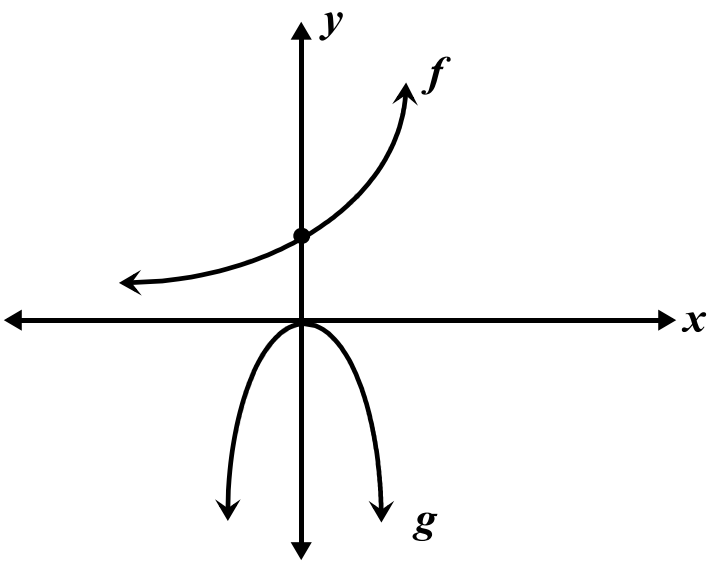
c.) Sketch the graph of *f*

d.) Write down the range of $y = -f(x)$

e.) Describe, in words, the transformation of *f* to *g* if $g(x) = \frac{-3}{x+1} - 2$

## Question 2

Sketched below are the graphs of $f(x)=3^x$ and $g(x)=-x^2$




a.) Write down the equation of the inverse of the graph of $f(x)=3^x$ in the form $f^{-1}(x)=.....$

b.) On a set of axes, draw the graph of the inverse of $f(x)=3^x$

c.) Write down the domain of the graph of $f^{-1}(x)$

d.) Explain why the inverse of the graph of $g(x)=-x^2$ is not a function.

e.) Consider the graph of $g(x)=-x^2$

i.) Write down a possible restriction for the domain of $g(x)=-x^2$ so that the inverse of the graph of *g* will now be a function.

ii.) Hence draw the graph of the inverse function

f.) Explain how, using the transformation of the graph of *f*, you would sketch the graphs of:

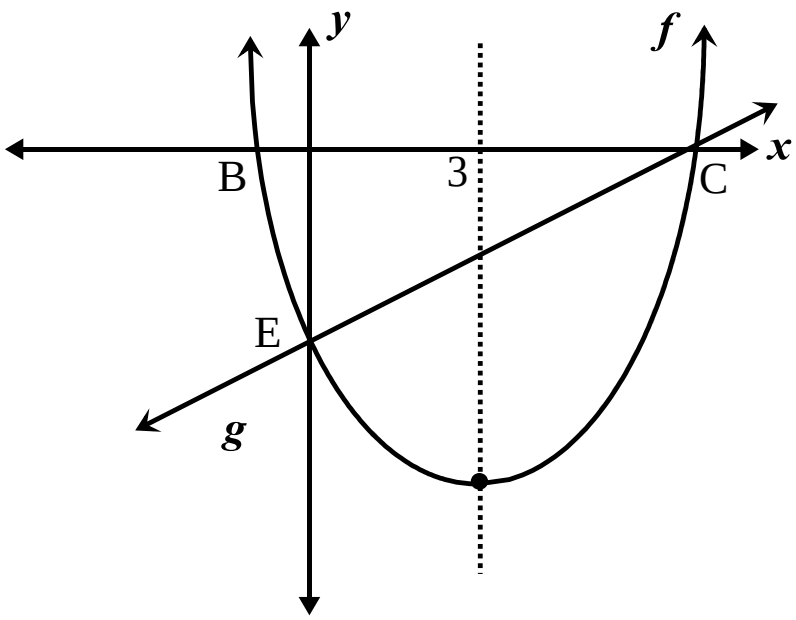
i.) $h(x)=-\log_3 x$

ii.) $p(x)=\left(\frac{1}{3}\right)^x+1$

g.) Sketch the graph of $p(x)=\left(\frac{1}{3}\right)^x+1$ on a set of axes.

## Question 3

A parabola $f$ intersects the $x$-axis at B and C and the $y$-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.




a.) Show that the coordinates of C are $(7\,;0)$

b.) Calculate the $x$-coordinate of B.

c.) Determine the equation of $f$ in the form $y = a(x-p)^2 + q$.

d.) Write down the equation of the graph of $h$, the reflection of $f$ in the $x$-axis.

e.) Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$

f.) Solve for $x$ if $f(x^2 - 2) = 0$

## Question 4

Consider the functions: $f(x) = 2x^2$ and $g(x) = \left(\frac{1}{2}\right)^x$

a.) Restrict the domain of $f$ in one specific way so that the inverse of $f$ will also be a function.

b.) Hence draw the graph of your new function $f$ and its inverse function $f^{-1}$ on the same set of axes.

c.) Write the inverse of $g$ in the form $g^{-1}(x) = .....$

d.) Sketch the graph of $g^{-1}$.

e.) Determine graphically the values of $x$ for which $\log_{\frac{1}{2}} x < 0$