CALCULUS I 06 MAY 2013

Lesson Description

In this lesson we:

- Look at the concept of limits
- Calculate the derivative from first principles
- Calculate the derivative using the rules of differentiation

Key Concepts

Calculus is the study of rates of change and motion. It is one of the most powerful mathematical tools ever invented and is used every day by mathematicians, physicists, engineers and indirectly by a number of other professions.

Calculus is divided into two parts:

1. Differential Calculus - rates of change
2. Integral Calculus - area (integration)

Real Life Applications of Calculus

- Biologists use differential calculus to determine the exact rate of growth in a bacterial culture when different variables such as temperature and food source are changed. This research can help increase the rate of growth of necessary bacteria, or decrease the rate of growth for harmful and potentially threatening bacteria.
- An architect will use integration to determine the amount of materials necessary to construct a curved dome over a new sports arena, as well as calculate the weight of that dome and determine the type of support structure required.
- A graphics artist uses calculus to determine how different three-dimensional models will behave when subjected to rapidly changing conditions. This can create a realistic environment for movies or video games.

Limits

When finding the limit of a function we look at what happens to the function value, as we get close to a specific x-value on the curve.

Example

Find: $\lim_{x \to 0} 3x+6$

The Gradient of a Curve at a Point

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example

Find the gradient of the curve:
$f(x) = x^2$ at (2,4)

First Principles

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ we call this the derivative of f at x

Find the derivative of the following from first principles:

1. $f(x) = x^2 + x$
2. $f(x) = \frac{1}{x}$

Rules for Differentiation

Derivative of a constant is zero
if $f(x) = k$ if k is a constant then $f'(x) = 0$

Power Rule

if $y = x^n$ then $y' = n.x^{n-1}$

If $f(x) = k.x^n$ then $f'(x) = k.nx^{n-1}$

Questions

Question 1

Find the derivative of:

a. $f(x) = 3x^2 + 8$

b. $y = 4x^3 + ½x^2 + x$

c. $f(x) = \sqrt{x} + 1/\sqrt{x}$

d. $y = -x^2(x - 2)$

e. $y = \frac{x^2 - 2x + 2}{x}$