CALCULUS III 20 MAY 2013

Lesson Description

In this lesson we:

- Look at Concavity
- Discuss Cubic Graphs

Key Concepts

Recap

Increasing Function

- y values increase as the x values increase.
- Derivative is positive (gradient of tangent is positive)

Decreasing Function

- y values decrease as the x values increase.
- Derivative is negative (gradient of tangent is negative)

The points where the derivative is equal to zero are called **stationary points**-the function is not increasing or decreasing at these points.

Concavity and Points of Inflection

The second derivative test helps you find the local maximum and minimum points of a function.

If f''(x) < 0 you have a local maximum (turning point)
If f''(x) > 0 you have a local minimum (turning point)
If f''(x) = 0 then no conclusion can be made about the stationary point.

Points of inflection can occur where f''(x) =0 at the point of inflection; function changes concavity.

Curve Sketching: $y = ax^3 + bx^2 + cx + d$

- Find the x and y intercepts
- Find the stationary points
- Investigate f''(x)
- Consider what happens to the function as $x \rightarrow \pm\infty$
- Sketch the graph

Questions

Question 1

$f(x) = x^3 - 12x^2 + 36x$

Find:

a.) The stationary points
b.) Determine the point(s) of inflection
c.) Determine where the function is concave up or concave down
d.) Sketch f(x)

Question 2

Sketch the graph $y = -x^3 + 4x^2 - x$

a.) Find the intercepts with the axes
b.) Find the maxima and minima and point of inflection
c.) Examine the behaviour of the graph as $x \to \pm\infty$

Challenge Question

Question 1

Given $g(x) = 2x^3 + 3x^2 + 4$

a.) Determine the x and y intercepts of g. (5)
b.) Determine the coordinates of the stationary points and state their natures. (5)
c.) Sketch g clearly labelling all relevant points. (4)
d.) Determine $g''\left(-\frac{1}{2}\right)$ and state the significance of this point. (3)
e.) Determine the equation of the tangent to g at $x = -2$. (3)
f.) Determine the coordinates of the point where the tangent in (e.) intersects g again. (4)
g.) If $f(x) = g(x) - 5$ write down the coordinates of the stationary points of f. (4)
h.) For which values of k does the equation $2x^3 + 3x^2 + 4 = k$ have three different x intercepts? (2)