CALCULUS IV 27 MAY 2013

Lesson Description

In this lesson we:

- Take a close look at calculus applications

Key Concepts

- Cubic graph applications
- Applications of maxima and minima and rates of change.

Maxima and Minima

Useful formulae for maxima and minima:

Volume = area of base x height

Total surface area = 2(area of base) + perimeter of base x height
[for a closed shape]

Rates of Change

A rate of change can be positive or negative.

Example

If the volume of air in a balloon is increasing and the radius of the spherical balloon is expanding, then the rate of change of volume with respect to the radius is positive. However, if air escapes from the balloon, then the rate of change is negative.

Note

s(t) represents an equation of motion (height, distance, displacement etc.) at time t.
s'(t) represents the speed or velocity at time t.

Questions

Question1

(Adapted from Feb/Mar 2012, Paper 1, Questions 9)

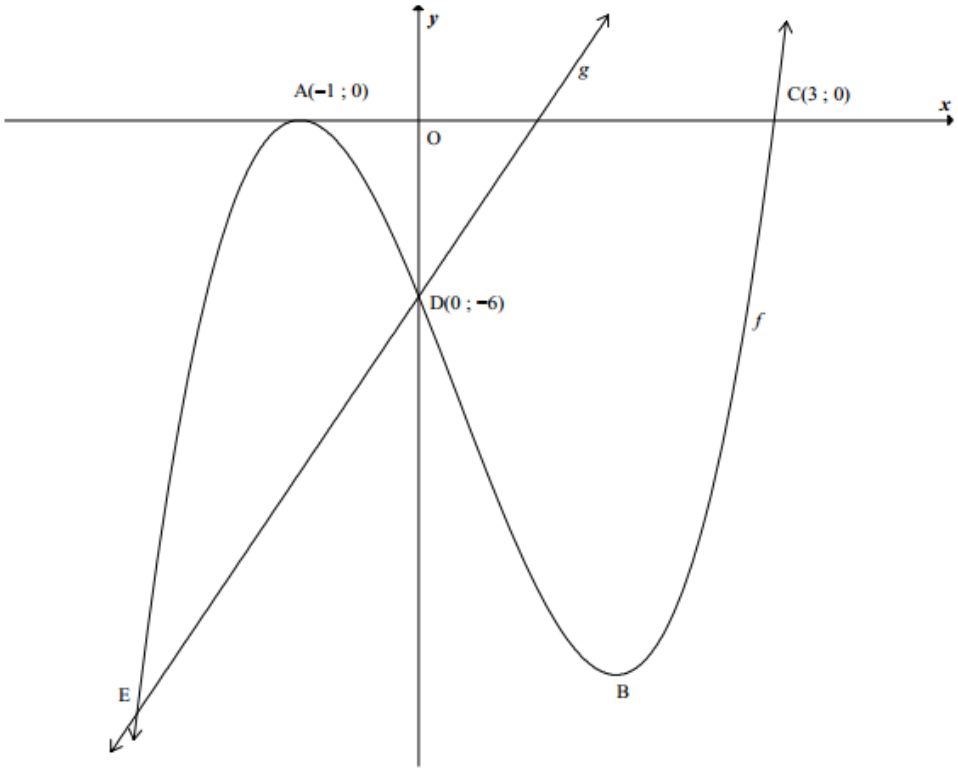
The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x-6$ are sketched below.

A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f.

The graph of f has turning points at A and B.

D(0 ; – 6) is the y-intercept of f.

E and D are points of intersection of the graphs of f and g.



a.) Show that a = 2; b = – 2 ; c = – 10 and d = – 6. (5)
b.) Calculate the coordinates of the turning point B. (5)
c.) h(x) is the vertical distance between f(x) and g(x), that is h(x) = f(x) – g(x).
Calculate x such that h(x) is a maximum, where x < 0. (5)

[15]

Question 2

The radius of the base of a circular cylindrical can is *x* cm, and its volume is 430 cm^3

a.) Determine the height of the can in terms of *x*
b.) Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x
c.) Determine the value of x for which the least amount of material is needed to manufacture such a can
d.) If the cost of the material is R500 per m^2 what is the cost of the cheapest can (labour excluded)?

Question 3

The volume of water in a tank is given by $V = 5+10t - t^2$
(V=Volume in m^3, t= time in minutes)

a.) Find the rate at which the volume is increasing when t=2
b.) At what time does the volume start decreasing?