REVISION: CALCULUS 10 JUNE 2013

Lesson Description

In this lesson we revise how to:

- calculate the derivative of a function from first principles
- calculate the derivative using the rules of differentiation
- find the equation of a tangent to a curve at a point
- sketch the graph of a cubic function

Key Concepts

Calculus is the study of rates of change and motion. It is one of the most powerful mathematical tools ever invented and is used every day by mathematicians, physicists, engineers and indirectly by a number of other professions.

Calculus is divided into two parts:

1. Differential Calculus - rates of change
2. Integral Calculus - area (integration)

In Grade 12, we introduce differential calculus as part of Paper 1.

Limits

When finding the limit of a function we look at what happens to the function value, as we get close to a specific x-value on the curve.

Example

Find: $\lim\limits_{x \to 0} 3x+6$

The Gradient of a Curve at a Point

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example

Find the gradient of the curve:
$f(x) = x^2$ at (2,4)

First Principles

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ we call this the derivative of f at x

Find the derivative of the following from first principles:

1. $f(x) = x^2 + x$
2. $f(x) = \frac{1}{x}$

Rules for Differentiation

Derivative of a constant is zero
if $f(x) = k$ if k is a constant then $f'(x) = 0$

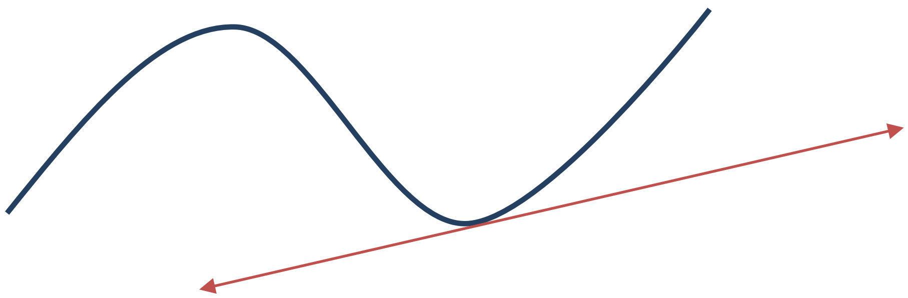
Power Rule

if $y = x^n$ then $y' = n.x^{n-1}$

If $f(x) = k.x^n$ then $f'(x) = k.nx^{n-1}$

What is a Tangent?

A tangent is a line that touches a curve at a point. Tangents are straight lines defined by $y = mx + c$.



A tangent to a curve will have the same gradient as the curve at the point of contact.

How do you find the gradient of a curve?
The gradient of a tangent to a curve is equal to the value of the first derivative at that point

Increasing Function

- y values increase as the x values increase.
- Derivative is positive (gradient of tangent is positive)

Decreasing Function

- y values decrease as the x values increase.
- Derivative is negative (gradient of tangent is negative)

The points where the derivative is equal to zero are called **stationary points**-the function is not increasing or decreasing at these points.

Concavity and Points of Inflection

The second derivative test helps you find the local maximum and minimum points of a function.

If $f''(x) < 0$ you have a local maximum (turning point)
If $f''(x) > 0$ you have a local minimum (turning point)
If $f''(x) = 0$ then no conclusion can be made about the stationary point.
Points of inflection can occur where $f''(x) = 0$ at the point of inflection; function changes concavity.

Curve Sketching: $y = ax^3 + bx^2 + cx + d$

- Find the x and y intercepts
- Find the stationary points
- Investigate $f''(x)$
- Consider what happens to the function as $x \rightarrow \pm\infty$
- Sketch the graph

Questions

Question 1

Find the derivative of:

a. $f(x) = 3x^2 + 8$

b. $y = 4x^3 + ½x^2 + x$

c. $f(x) = \sqrt{x} + 1/\sqrt{x}$

Question 2

Determine the equation of the tangent to the graph $y = (2x + 1)^2(x + 2)$ at $x = ¾$

Question 3

$f(x) = x^3 - 12x^2 + 36x$

Find:

a.) The stationary points
b.) Determine the point(s) of inflection
c.) Determine where the function is concave up or concave down
d.) Sketch f(x)