TRIGONOMETRY: 2-D AND 3-D TRIANGLES 12 AUGUST 2013

Lesson Description

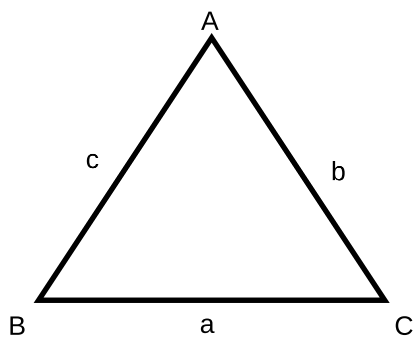
In this lesson, we:

- Apply the sine, cosine and area rules to determine unknown sides and angles in both 2 dimensional triangles
- Then, apply the sine, cosine and area rules to determine the unknown sides and angles of 3 dimensional triangles.

Key Concepts

- The area, sine and cosine rules are usually applied when we are required to find the missing sides and angles of triangles that are typically **not** right angle triangles.

The Area Rule

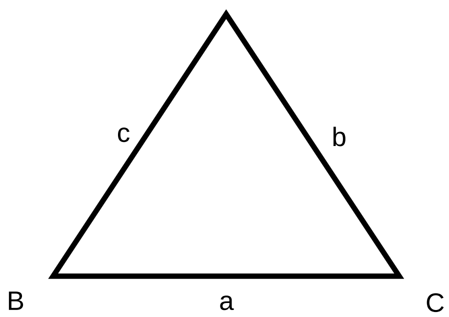


Area Δ ABC = $\frac{1}{2}bc \sin A$

Or $= \frac{1}{2}ac\ sinB$

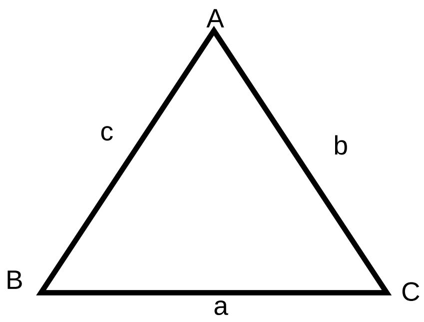
Or $= \frac{1}{2}ab\ sinC$

The Sine Rule



$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

The Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\ cosA$$

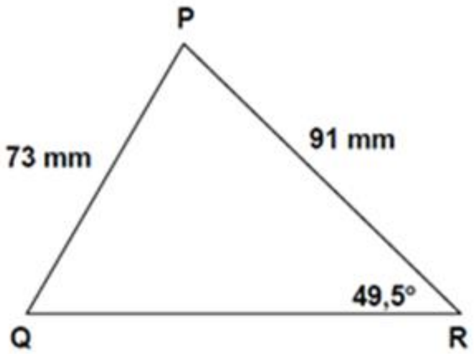
$$b^2 = a^2 + c^2 - 2ac\ cosB$$

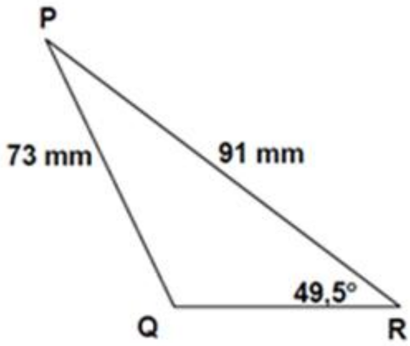
$$c^2 = a^2 + b^2 - 2ac\ cosC$$

Questions

Question 1

ΔPQR has the following dimensions: PR = 91 mm, PQ = 73 mm and R = 49,5. Two triangles are shown below with these dimensions.



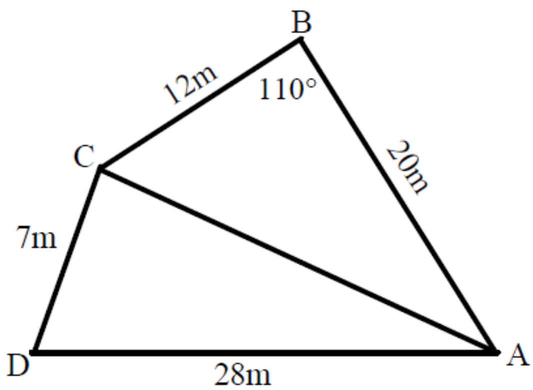


a.) Find the two possible sizes of Q correct to 1 decimal place.
b.) Hence find the greater length of QR, to 1 decimal place.

Question 2

A piece of land has the form of a quadrilateral ABCD with AB = 20m, BC 12m, CD = 7m and AD = 28 m. B = 110°.

The owner decides to divide the land into two plots be erecting a fence from A to C.

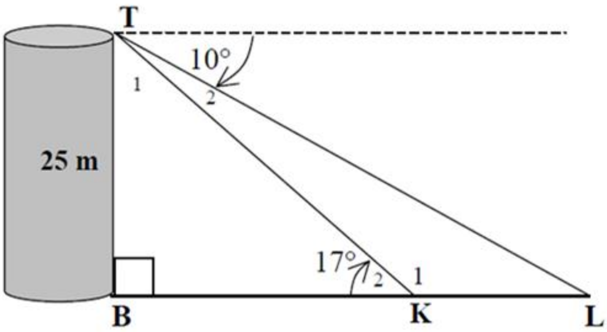


a.) Calculate the length of the fence AC.
b.) Calculate the size of BAC correct to the nearest degree.
c.) Calculate the size of D, correct to the nearest degree.
d.) Calculate the area of the entire piece of land ABCD, correct to one decimal place.

Question 3

The diagram below is a representation of a 25m vertical observation tower TB and two cars K and L on a road. The angle of depression T to car L is 10°. The angle of elevation from car K to the top of the tower is 17°.

B, K and L lie in a straight line and lie on the same horizontal place as the base of the tower TB.

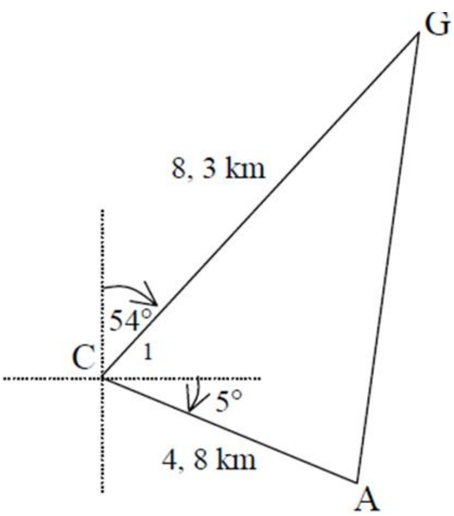


a.) Calculate the size of L.
b.) Calculate the length of KT.
c.) Hence calculate the distance between the two cars.

Question 4

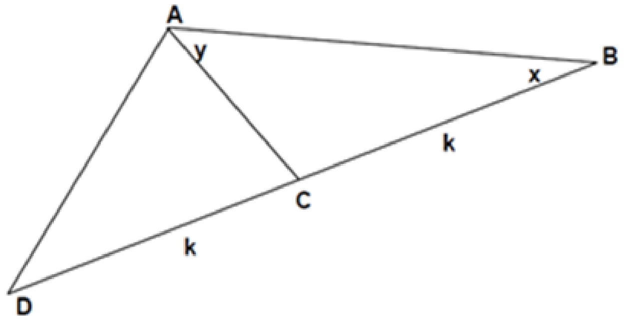
A game ranger G is 8,3 km from control centre, C, at a bearing of 54° east when he receives a call that there is an injured antelope, A, that needs attention.

The antelope is located 4,8 km at a bearing of 5° south of east from the control centre. The diagram below is a representation of the above-mentioned situation.



a.) Calculate how far the game ranger is from the injured antelope.
b.) Calculate the area of ΔGCA.

Question 5



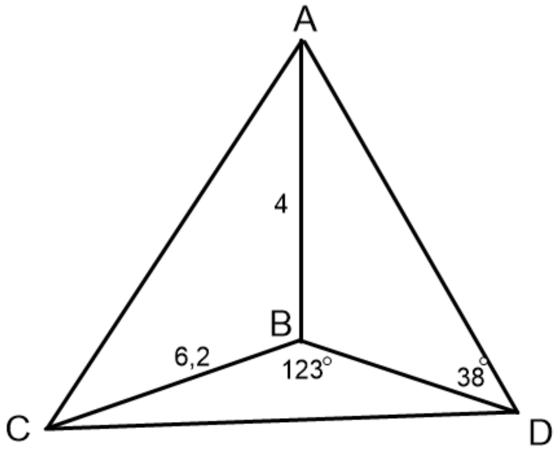
BC = CD = k.

$A\hat{B}C = x$ and $C\hat{A}B = y$

Prove that area ΔADC = $\dfrac{k^2.sinx.\sin(x+y)}{2siny}$

Question 6

C and D are two points in the same horizontal plane as B, the foot of a vertical pole, AB. The height of the pole is 4 m. AC and AD are cables. The angle of elevation of A from D is 38°, DBC = 123° and BC = 6,2 m.



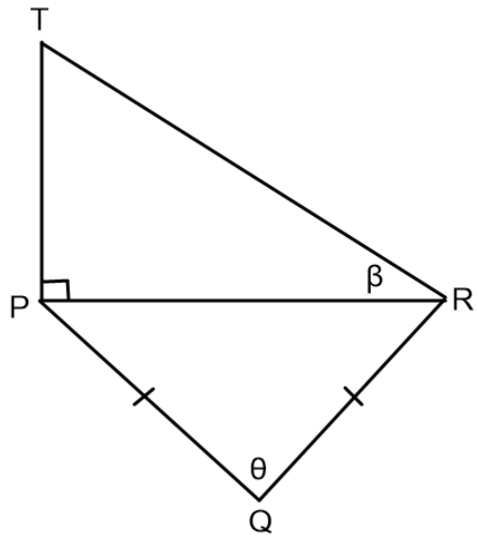
a.) The length of DB.
b.) The length of CD.
c.) The area of ΔBCD

Question 7

P, Q and R are three points in the same horizontal plane. PT is a vertical pole. The angle of elevation from R to T is β.

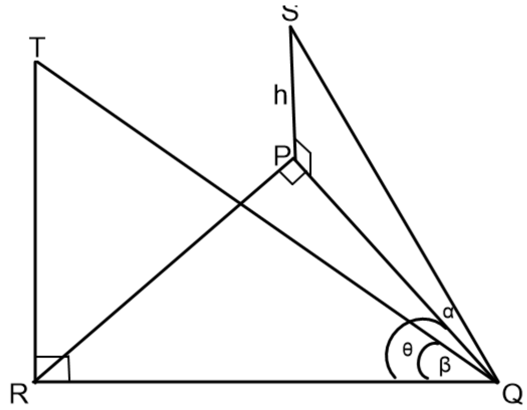
Q is equidistant from P and R and PQR = θ

Prove that PT = PQtan $\beta\sqrt{2(1-cos\theta)}$



Question 8

From a point Q at a navy base, two passing jets, S and T, are observed. P and R are points on the ground directly beneath S and T, respectively, and in the same horizontal plane as Q. The angles of elevation of S and T from Q are α and β respectively. In addition, PQR = 90° and PQR = θ. The height of S above the ground is given as h meters.



a.) Prove that TR = $\frac{htan\beta}{tan\alpha cos\theta}$

b.) Calculate the height of T above the ground if h = 145m, θ = 67,3°, α = 27,7° and β = 49,4°