TRIGONOMETRY: 2-D AND 3-D TRIANGLES II 19 AUGUST 2013

Lesson Description

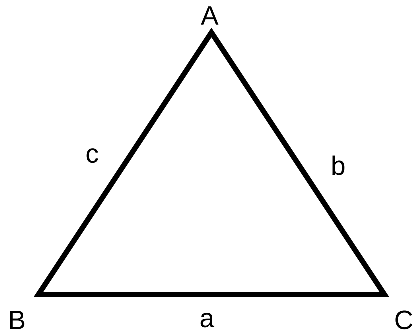
In this lesson, we:

- Apply the sine, cosine and area rules to determine the unknown sides and angles of 3 dimensional triangles.
- Apply compound and double angle formulas in order to prove the required sides and angles of the triangle.

Key Concepts

- The area, sine and cosine rules are usually applied when we are required to find the missing sides and angles of triangles that are typically **not** right angle triangles.

The Area Rule

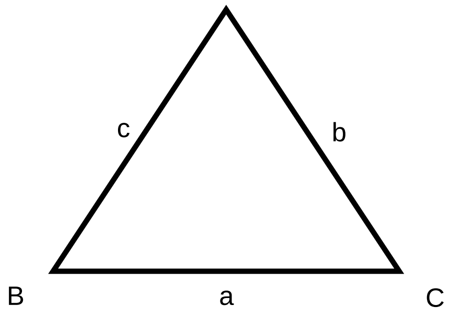


Area Δ ABC $= \frac{1}{2}bc\sin A$

Or $= \frac{1}{2}ac\ sinB$

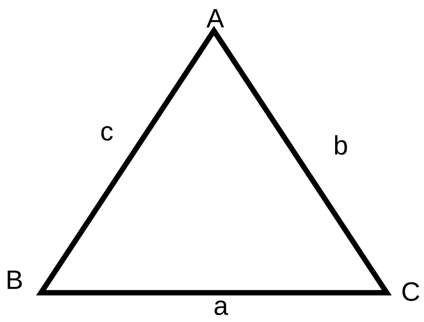
Or $= \frac{1}{2}ab\ sinC$

The Sine Rule



$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

The Cosine Rule



$a^2 = b^2 + c^2 - 2bc\ cosA$

$b^2 = a^2 + c^2 - 2ac\ cosB$

$c^2 = a^2 + b^2 - 2ac\ cosC$

Compound Angle Formulas

cos(a + b) = cosa cosb - sina sinb

cos(a - b) = cosa cosb + sina sinb

sin(a + b) = sina cosb + cosa sinb

sin(a - b) = sina cosb - cosa sinb

Double Angle Formulas

sin2a = 2 sina cosa

cos2a = $\cos^2$ a - $\sin^2$ a

or = $2\cos^2$ a - 1

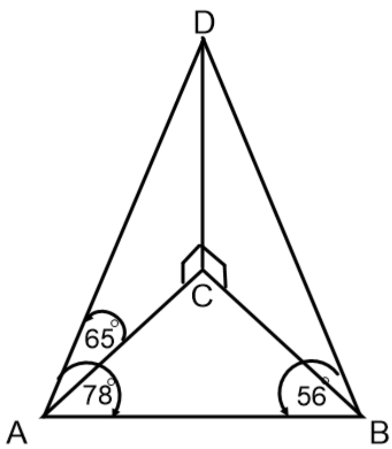
or = 1 - $2\sin^2$a

Questions

Question 1

A, B and C are 3 points in the same horizontal plane and AB is 53m in length. CD is a vertical tower and the angle of elevation of D from A is 65.

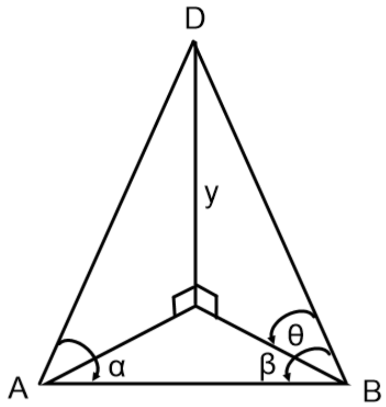
DAB = 78 and DBA= 56.



a.) Determine the size of ADB
b.) The length of AD
c.) The height of tower CD

Question 2

In the diagram below, D is a point vertically above C. DC is y meters in length. The angle of elevation of D from B is? Angle DAB = a and DBA = ß.



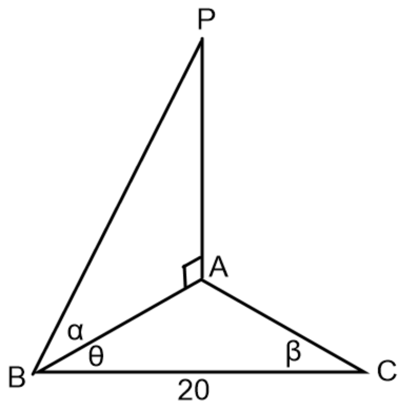
a.) Determine the length of DB in terms of y and θ

b.) Show that AB = $\dfrac{y\sin(\alpha+\beta)}{\sin\theta\sin\alpha}$

Question 3

In the figure below A, B and C are in the same horizontal plane. P is a point vertically above A . B and C are 20 units apart. The angle of elevation from B to P is a.

ABC = α and ACB = β



a.) Prove that AP = $\dfrac{20\sin\beta\tan\alpha}{\sin(\theta+\alpha)}$

b.) Given that AB = AC, show that AP = $\dfrac{10\tan\alpha}{\mathrm{Cos}\beta}$

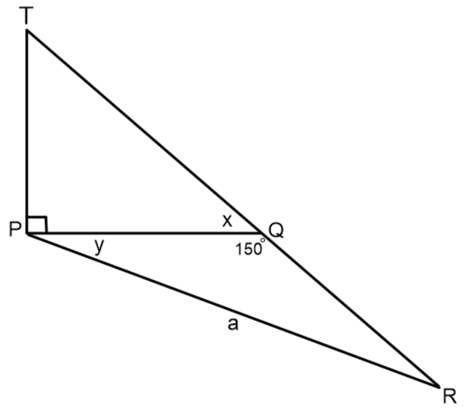
c.) If AB = AC, determine AP if β = 55 and α = 75

Question 4

TP is a tower. Its foot, P, and the points Q and R are on the same horizontal plane. From Q the angle of elevation to the top of the building is x. It is further given that

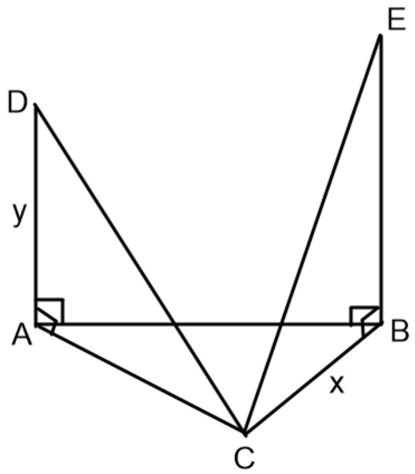
PQR = 150 , QPR = y and the distance between P and R is a.

Prove that TP = a tanx (cosy + $\sqrt{3}$siny)



Question 5

A telephone cable is to be erected between two cliff sides AD and BE. An engineer stands at point C in the same horizontal plane as the foot of the cliffs. He measures the angle of E from C and D to be θ and α respectively. Cliff DA is y meters in height and C is x meters from the foot of cliff BE.



Show that the length of the telephone cable is $\frac{x\,tan\theta - y}{sin\alpha}$