REVISION: TRIGONOMETRY & ANALYTICAL GEOMETRY

16 SEPTEMBER 2013

Lesson Description

In this lesson, we:

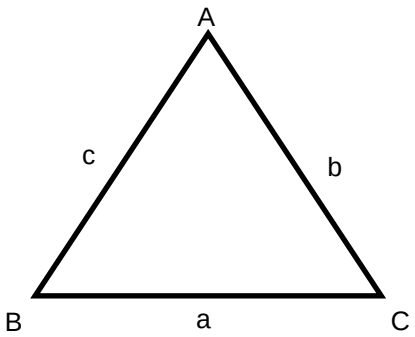
- revise how to apply the sine, cosine and area rules to determine unknown sides and angles in both 2 & 3dimensional triangles
- revise and apply the concepts of distance formula, mid-point formula, gradient, inclination to and the equation of a circle with centre off the origin, to solve problem on a Cartesian plane

Key Concepts

Trigonometry

- The area, sine and cosine rules are usually applied when we are required to find the missing sides and angles of triangles that are typically **not** right angle triangles.

The Area Rule

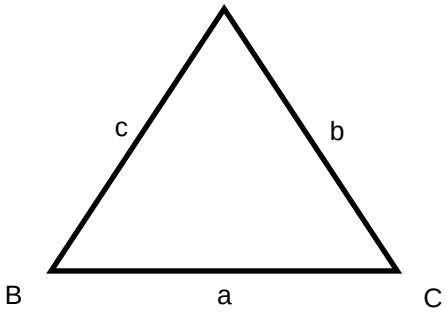


Area Δ ABC = $\frac{1}{2}bc\sin A$

Or $= \frac{1}{2}ac\ sinB$

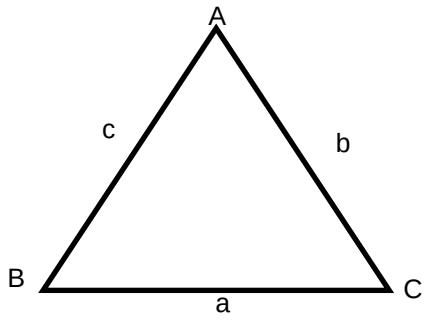
Or $= \frac{1}{2}ab\ sinC$

The Sine Rule



$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

The Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\ cosA$$

$$b^2 = a^2 + c^2 - 2ac\ cosB$$

$$c^2 = a^2 + b^2 - 2ac\ cosC$$

Compound Angle Formulas

cos(a + b) = cosa cosb - sina sinb

cos(a - b) = cosa cosb + sina sinb

sin(a + b) = sina cosb + cosa sinb

sin(a - b) = sina cosb - cosa sinb

Double Angle Formulas

sin2a = 2 sina cosa

cos2a = $\cos^2$ a - $\sin^2$ a or = 2$\cos^2$ a – 1 or = 1 - 2$\sin^2$a

Analytical Geometry

The Distance Formula

$$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

The Mid-Point Formula

$$\frac{x_1 - x_2}{2} ; \frac{y_1 - y_2}{2}$$

The Gradient of a Line

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Inclination of a Line

$$tan\theta = m$$

Equation of Circle with Centre (m ; n)

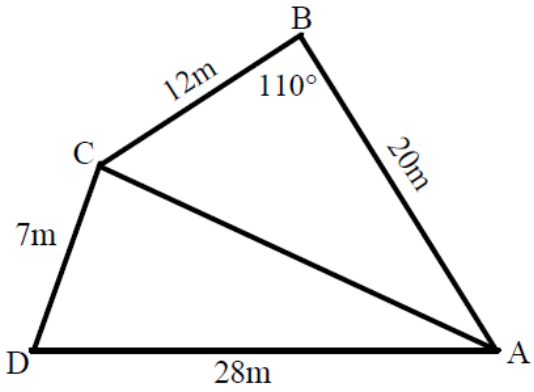
$$(x - m)^2 + (y - n)^2 = r^2$$

Questions

Question 1

A piece of land has the form of a quadrilateral ABCD with AB = 20m, BC 12m, CD = 7m and AD = 28 m. B = 110°.

The owner decides to divide the land into two plots be erecting a fence from A to C.

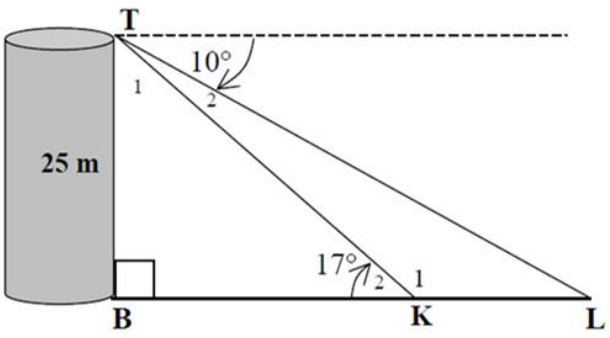


a.) Calculate the length of the fence AC.
b.) Calculate the size of BAC correct to the nearest degree.
c.) Calculate the size of D, correct to the nearest degree.
d.) Calculate the area of the entire piece of land ABCD, correct to one decimal place.

Question 2

The diagram below is a representation of a 25m vertical observation tower TB and two cars K and L on a road. The angle of depression T to car L is 10°. The angle of elevation from car K to the top of the tower is 17°.

B, K and L lie in a straight line and lie on the same horizontal place as the base of the tower TB.

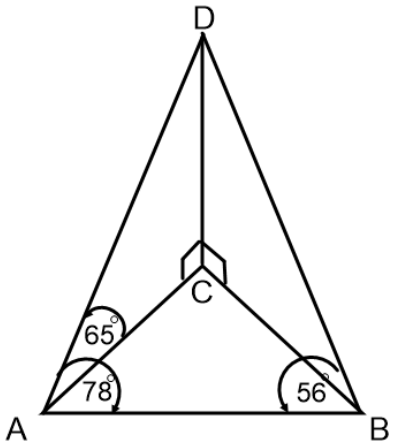


a.) Calculate the size of L.
b.) Calculate the length of KT.
c.) Hence calculate the distance between the two cars.

Question 3

A, B and C are 3 points in the same horizontal plane and AB is 53m in length. CD is a vertical tower and the angle of elevation of D from A is 65.

DAB = 78 and DBA= 56.



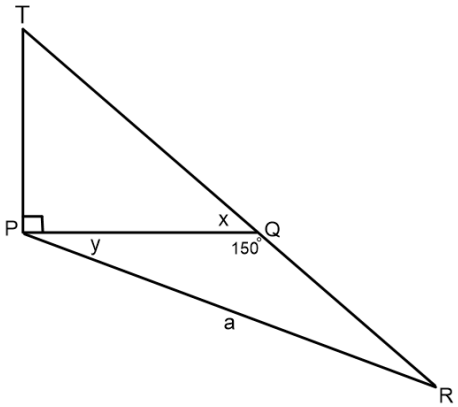
a.) Determine the size of ADB
b.) The length of AD
c.) The height of tower CD

Question 4

TP is a tower. Its foot, P, and the points Q and R are on the same horizontal plane. From Q the angle of elevation to the top of the building is x. It is further given that

PQR = 150 , QPR = y and the distance between P and R is a.

Prove that TP = a tanx (cosy + $\sqrt{3}$siny)



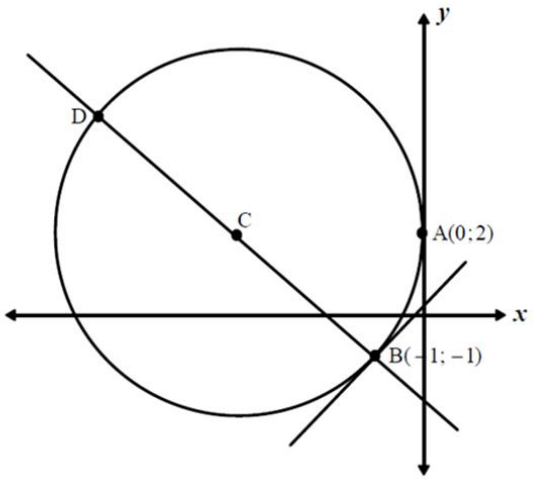
Question 5

Calculate the value of k if the points A(6;5), B(3;2) and C(2k ; k+4) are collinear.

Question 6

In the diagram below, a circle centre C touches the y-axis at A(0;2). A straight line with equation

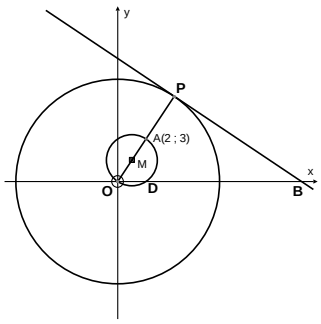
3x + 4y = -7 cuts the circle at B(-1; -1) and D.



a.) Determine the equation of the tangent to the circle at B
b.) Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$
c.) Determine the coordinates of D

Question 7

A(2 ; 3), the midpoint of radius OP, lies on the circumference of the smaller circle with diameter OA.



Write down:

a.) The co-ordinates of P.
b.) The equation of the larger circle.
c.) The equation of the smaller circle.

Question 8

A circle is represented by the equation: $x^2+2x+y^2-4y-5=0$

a.) A transformation moves every point 2 units to the left and 4 units up. Determine the equation of the new circle after the transformation.
b.) Does the origin lie within the new circle? Give a reason for your answer.