

PROJECTILE MOTION: EQUATIONS AND GRAPHS

12 FEBRUARY 2013

Lesson Description

In this lesson we will

- Learn that all projectiles fall freely under gravity and accelerate at “g” whether they are moving up or down.
- Use simple equations of motion to describe their motion.
- Look at using graphs of displacement, velocity and acceleration vs. time to show the flight of projectiles.
- Discuss using a structured approach to solve projectile motion questions and graphs.

Key Concepts

- Gravitational acceleration (acceleration that applies to ALL projectiles)
- Trajectory (the path/motion of a projectile)
- Apex/apogee (the maximum height achieved by a projectile)

When objects are allowed to fall or are thrown, shot or kicked up into the air, we call them projectiles. We will be studying only the flight of projectiles in this section.

One thing that needs to be clear in our minds before we go forward is what gravitational acceleration is.

Gravitational Acceleration Calculations ($g = 9,8 \text{ m}\cdot\text{s}^{-2}$)

When an object is dropped, it will accelerate due to the force of gravity. Galileo Galilei famously proved that ALL objects accelerate at the same rate. This means that mass does not change the way that gravity accelerates an object.

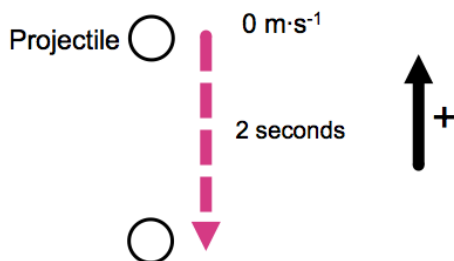
This means that an object of 5 kg will accelerate at the same rate as another object of mass 50 g. We can drop both objects and the result will be that they strike the ground at exactly the same time, with the same final velocity.

Let's see study how this gravitational acceleration affects objects which are dropped.

If we drop an object from a particular height – this means that gravity is the ONLY force acting on it. It accelerates at $9,8 \text{ m}\cdot\text{s}^{-2}$. We call this value g.

Example 1:

Calculate how fast a ball will be moving it is allowed to fall for 2 seconds from a stationary point.

Step 1: Diagram and Direction

Notice how the direction that we consider positive is marked with an arrow and a “plus” sign?

Step 2: Tabulate

It's important to know what information we have and what we don't have. Some of the numbers not directly given to us so we must READ carefully.

$$v_i = 0 \text{ m} \cdot \text{s}^{-1}$$

$$v_f = ?$$

$$\Delta t = 2 \text{ s}$$

$$a = -9.8 \text{ m} \cdot \text{s}^{-2}$$

$$\Delta y = \dots$$

Notice how acceleration due to gravity is down, therefore making it a negative vector? Down means negative if we follow our own diagram.

Step 3: Equation

We now choose an equation with the information that we HAVE and the information that we NEED.

Tip: we don't need displacement, nor are we asked it – so look for the equation WITHOUT it.

$$v_f = v_i + a\Delta t$$

$$v_f = (0) + (-9.8)(2)$$

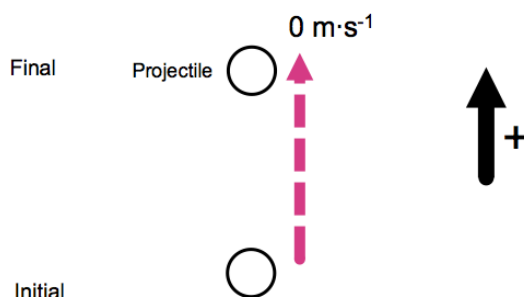
$$v_f = -19.6 \text{ m} \cdot \text{s}^{-1}$$

$$v_f = 19.6 \text{ m} \cdot \text{s}^{-1} \text{ downward}$$

Notice how the negative answer must be re-written so that direction is included. The answer was negative – meaning that the ball was moving downwards after 2 seconds. This is expected. The value also makes sense because it increases in velocity at $9.8 \text{ m} \cdot \text{s}^{-1}$ every second.

Example 2:

Calculate how far up a ball will rise if it is thrown upwards at $19,6 \text{ m}\cdot\text{s}^{-1}$?

Step 1: Diagram and Direction

Notice how the direction that we consider positive is marked with an arrow and a “plus” sign.

Also, notice that when a object reached it's maximum height – the VELOCITY becomes zero.

Step 2: Tabulate

It's important to know what information was have an don't have. Some of the numbers not directly given to us so we must READ carefully.

$$v_i = +19,6 \text{ m}\cdot\text{s}^{-1}$$

$$v_f = 0 \text{ m}\cdot\text{s}^{-1}$$

$$\Delta t = \dots$$

$$a = -9,8 \text{ m}\cdot\text{s}^{-2}$$

$$\Delta y = ?$$

Notice how gravitation acceleration is still negative and DOWNWARDS. If acceleration is in the opposite direction to motion the object will slow down. Think about what happens when you throw a ball up: it slows down and stops.

Step 3: Equation

We now choose an equation with the information that we HAVE and the information that we NEED.

Tip: we don't need time nor are we asked it – so look for the equation WITHOUT it.

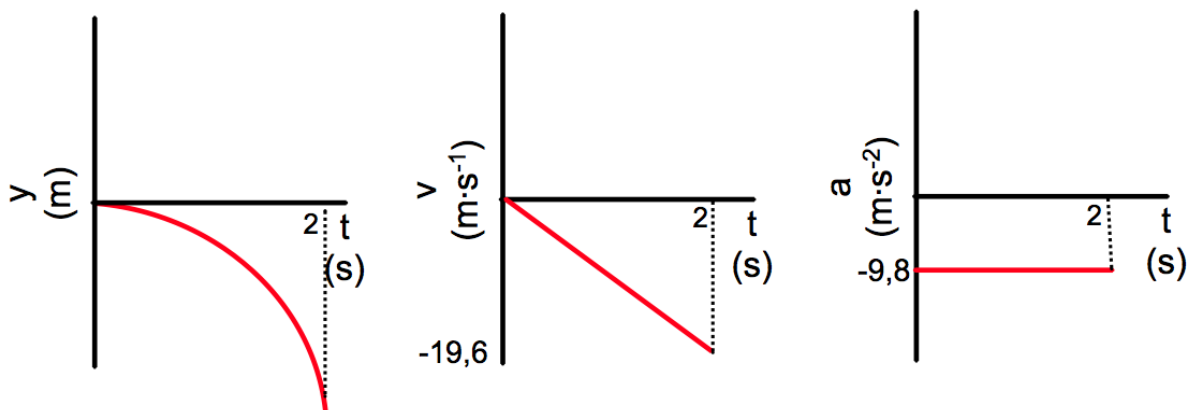
$$v_f^2 = v_i^2 + 2a\Delta x$$

$$(0)^2 = (19,6)^2 + 2(-9,8)\Delta x$$

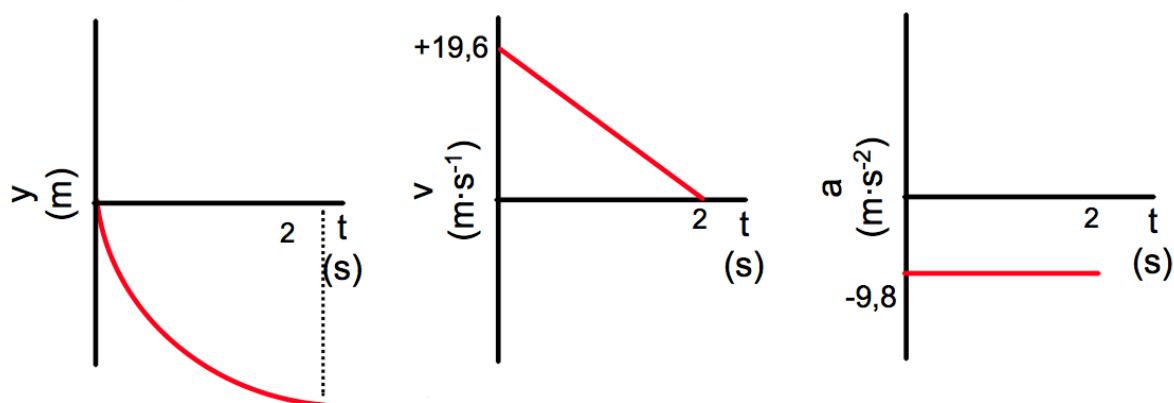
$$\Delta x = 19,6 \text{ m upward}$$

Graphs of Projectiles

Example 1:



Example 2:



Notice how the acceleration for both examples shows a downward acceleration. Challenge: see if you can draw the graphs for a ball thrown up and caught again at the same height. (Hint – final displacement will be zero)

Terminology

Projectile: an object which moves without touching other objects. Under only gravity's influence.

Gravitational acceleration: the acceleration of all objects in freefall, whether moving up or down, towards the earth.

Freefall: Motion of an object under the influence of only gravity as a force.

Initial: at the beginning.

Final: at the end.

Demonstration

To prove that objects accelerate due to gravity at the same rate, you can do a simple test.

Dropping objects is often too quick and it's difficult to tell which object hit the ground first so doing a "roll test" is often a better alternative.

When two round objects are rolled down the same incline, they also arrive at the bottom simultaneously. If you roll a big marble and a small marble down the same incline – they will both reach the bottom at the same time!

Questions

Question 1

(DOE March 2009 Question 5)

A supervisor, 1,8 m tall, visits a construction site. A brick resting at the edge of a roof 50 m above the ground suddenly falls. At the instant when the brick has fallen 30 m the supervisor sees the brick coming down directly towards him from above.

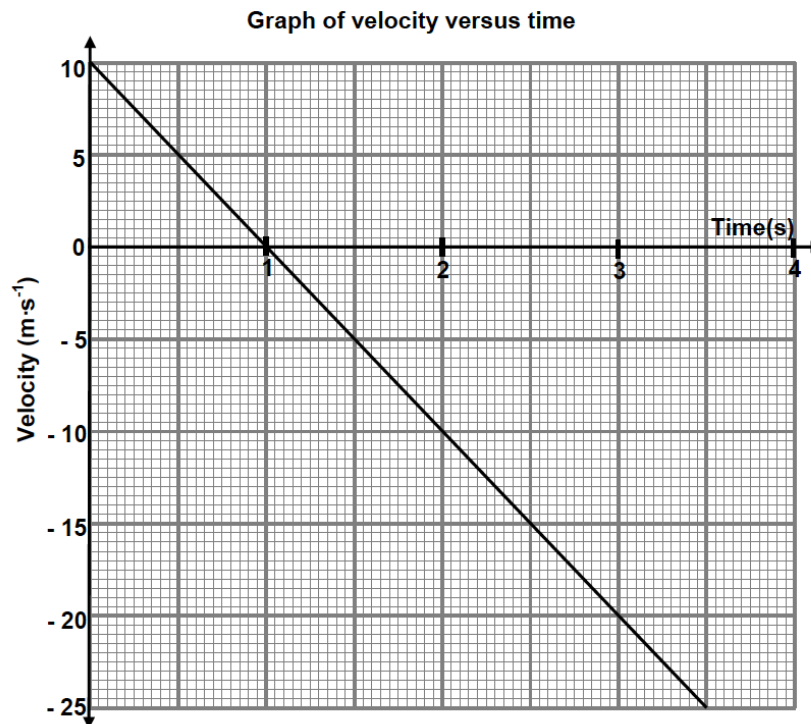
Ignore the effects of friction and take the downwards motion as positive.

- a.) Calculate the speed of the brick after it has fallen 30 m. (3)
 - b.) The average reaction time of a human being is 0,4 s. With the aid of a suitable calculation, determination whether the supervisor will be able to avoid being hit by the brick. (6)
- (9)**

Question 2

(DOE Nov 2008 Question 6)

A boy stands at the edge of a high cliff. He throws a stone vertically upwards with an initial velocity of $10 \text{ m}\cdot\text{s}^{-1}$. The stone strikes the ground at a point below the cliff after 3,5 s. The velocity-time graph below was obtained from *measurements* made during the motion of the stone.



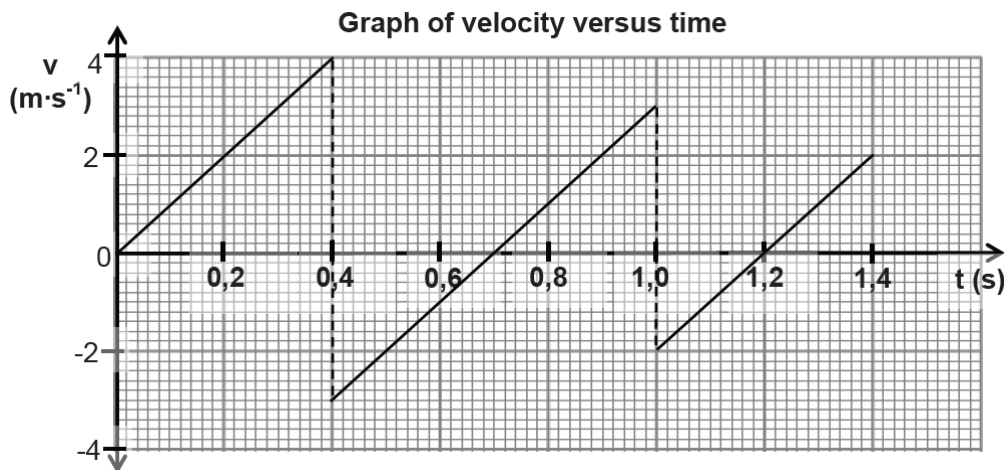
Use the information on the graph to answer the following questions:

- Calculate the acceleration of the stone between times $t = 2 \text{ s}$ and $t = 3 \text{ s}$. (3)
 - At which time(s) is the stone moving at a speed of $5 \text{ m}\cdot\text{s}^{-1}$? (2)
 - After how many seconds does the stone reach its highest point? (1)
 - Determine the height of the cliff from which the stone was thrown. (4)
 - Using the top of the cliff as the initial position of the stone, sketch the position-time graph (displacement-time graph) for the motion of the stone **from its highest point until it reaches the ground**. Only indicate relevant time values on the x-axis. (3)
- (13)**

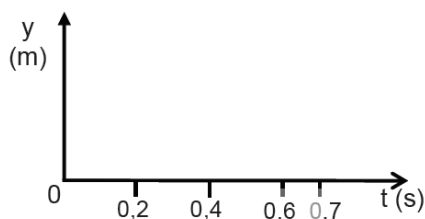
Question 3

(DOE November 2009 Question 4)

A ball is released from a certain height. The velocity-time graph below represents the motion of the ball as it bounces vertically on a concrete floor. The interaction time of the ball with the floor is negligibly small and is thus ignored.



- Describe the changes, if any, in velocity and acceleration of the ball from $T = 0$ s to $t = 0,4$ s. (4)
- Without using the equations of motion, calculate the height from which the ball has been dropped initially. (4)
- Copy the set of axes below.



Use the given velocity versus time graph for the motion of the ball to sketch the corresponding position-time graph for the time interval 0 s to 0,7 s. (3)

Links

- <http://www.mindset.co.za/learn/xtra>
- <http://www.education.gov.za/Examinations/PastExamPapers/tabid/351/Default.aspx>
- http://en.wikipedia.org/wiki/Projectile_motion