WORK AND ENERGY 26 FEBRUARY 2013

Lesson Description

In this lesson we:

- Learn that objects can have potential energy as a result of position and kinetic as a result of motion.
- Learn that work is the transfer of energy from one form into another.
- Use equations to describe work and energy.
- Use work and energy concepts to solve exam-type problems.

Key Concepts

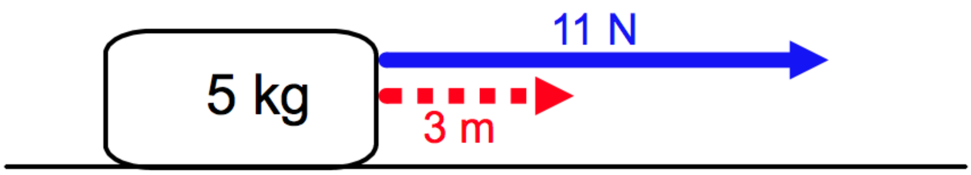
- Potential Energy is a result of the object's position relative to a force like gravity
- Kinetic Energy is the result of the object being in motion.
- Mechanical Energy is the sum of both Kinetic and Potential Energy.
- Work changes the amount of energy in an object when a force acts on the object over a distance.

Work

Work is done when a force moves an object over a distance.

It is very important to note that only when force and displacement act in the same plane, is work done.

However, from Grade 11, we know that forces can act parallel to the plane or perpendicularly to it. So let's use the example of an object pulled along by a force of 11 N over a distance of 3 m. For this example – let us have the force acting in the same direction as the displacement.



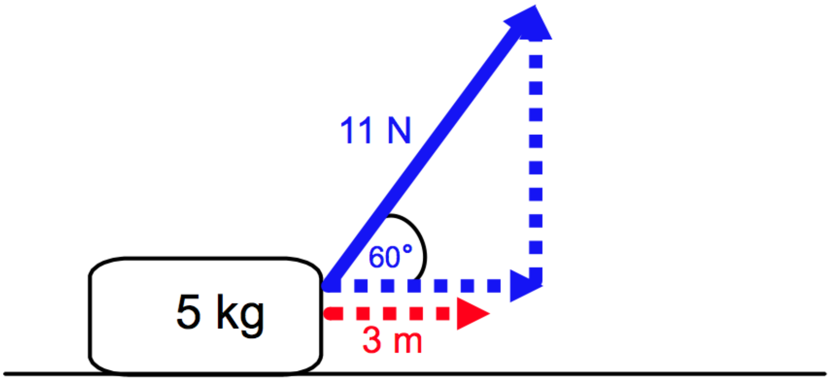
To calculate the work done is simple, we multiply the force applied and the displacement.

$$W = F_{\parallel}\Delta x$$

$$W = (11)(3)$$

$$W = 33\ \text{J}$$

What about if the force is pulled at 60° to the surface? Now we'd need to find out how much of the force is parallel to the surface – now we work with components. We need the parallel component of the applied force:



$$F_{\parallel} = F\cos\theta$$

$$F_{\parallel} = 11\cos 60°$$

$$F_{\parallel} = 5{,}5 \text{ N parallel to surface}$$

Now, we can multiply the force parallel to the displacement and get the work done:

$$W = F_{\parallel}\Delta x$$

$$W = (5{,}5)(3)$$

$$W = 16{,}5 \text{ J}$$

This is vewry complex and there is an easier way. The formula shown below works out the component of force for you and mulitplies it by the displacement!

$$W = F\Delta x\cos\theta$$

$$W = (11)(3)\cos 60°$$

$$W = 16{,}5 \text{ J}$$

Note: the angle is the one between the applied force and the displacement.

Potential and Kinetic Energy

Potential Energy is energy given to an object when it is moved against a force, like gravity. So, when an object is raised from the ground, we give it gravitational potential energy.

The equation describing this type of energy is:

$$U = E_p = mgh$$

- Notice how "mg" describes the force of gravity?
- Notice how "h" is actually vertical displacement against gravity?

The equation describing kinetic energy also depends on the objects mass but now, work is done to make the object go **faster**.

$$K = E_k = \frac{1}{2}mv^2$$

Together, we call these quantities **mechanical energy**.

In Grade 10, we saw that in an isolated system, *the total mechanical energy of a system will remian constant*.

But, now that we're in Grade 12 – we introduce the *Work-Energy Theorem*.

The Work-Energy Theorem

Remember:

- When a force is applied to an object while it moves, work is done.
- When there is a resultant/net force on the object, that object will accelerate
- The "extra" force, or net force does work that results in a change in the object's kinetic energy.

So, the equation looks like the one below:

$$W_{net} = \Delta K = \Delta E_k$$

The W_{net} is simply, the work done by all the forces acting on the object:

- If the force increases the energy of the object, by acting in the same direction as the movement, W_{net} becomes larger;
- If the force acts against the object moving, W_{net} becomes smaller.

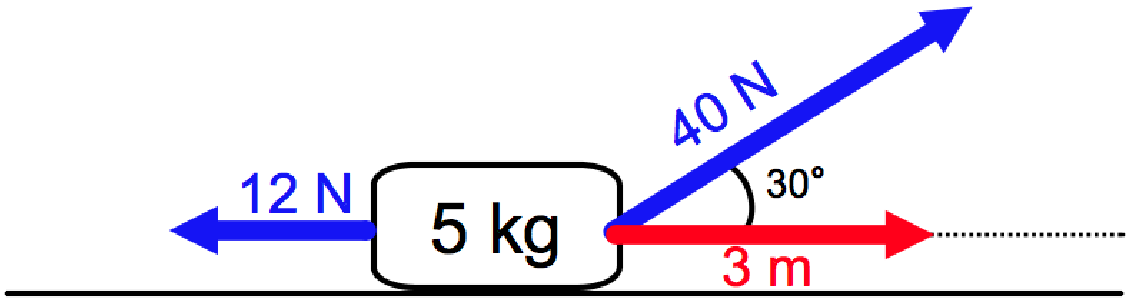
Example:

An object is pulled at at angle of 30° to the surface with a force of 40 N. The frictional force experienced by the object is 12 N. The object travels a distance of 3 m under these conditions.

1) What is the change in kinetic energy of this object.
2) If it was originally travelling at 1 $m{\cdot}s^{-1}$, calculate its final velocity after travelling the 3 m.

Solution:

Always draw a picture to show the information.



1) Let's find the work done by:

The applied force.

$$W_{app} = F_{app}\Delta x\cos\theta$$

$$W_{app} = (40)(3)\cos 30°$$

$$W_{app} = 103{,}92\ \text{J}$$

The frictional force.

$$W_{friction} = F_{friction}\Delta x\cos\theta$$

$$W_{friction} = (12)(3)\cos 180°$$

$$W_{friction} = -36\ \text{J}$$

Notice how frictional work is negative? This is because it decreased the energy of the object. The angle between the force (friction) and the displacement is 180**°.**

Now for the Work-Energy theorem:

$$W_{net} = \Delta K$$

$$W_{friction} + W_{app} = \Delta K$$

$$(-36) + (+103{,}92) = \Delta K$$

$$\Delta K = 67{,}92\ \text{J}$$

2) If it was originally travelling at 1 m·s^{-1}, calculate its final velocity after travelling the 3 m.

$$\Delta K = K_f - K_i$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$67{,}92 = \frac{1}{2}(5)v_f^2 - \frac{1}{2}(5)(1)^2$$

$$v_f = 5{,}12 \text{ m·s}^{-1}$$

Terminology

Force: a push or a pull on an object.

Potential energy: energy given to an object as a result of it's position relative to a force.

Freefall: energy of an object relative to its velocity.

Work: the change of energy from one form into another.

Work-energy theorem: the work doen by a net force on an object is equal to the change in its kinetic energy.

Demonstration

When a weight is swung from the end of a string, it experiences gravity and friction while it swings.

When it falls to its lowest level, it shows it's maximum kinetic energy. When the pendulum is left to swing for a while, you'll notice that it is swinging slower and slower.

The frictional force is causing a decrease in the amount of kinetic energy in the weight as it swings.

This shows that the work done by the friction is directly related to the change in the kinetic energy.

Questions

Question 1

(Adapted from DOE Nov 2012 Question 2)

A car moves up a hill at CONSTANT speed. Which ONE of the following represents the work done by the weight of the car as it moves up the hill?

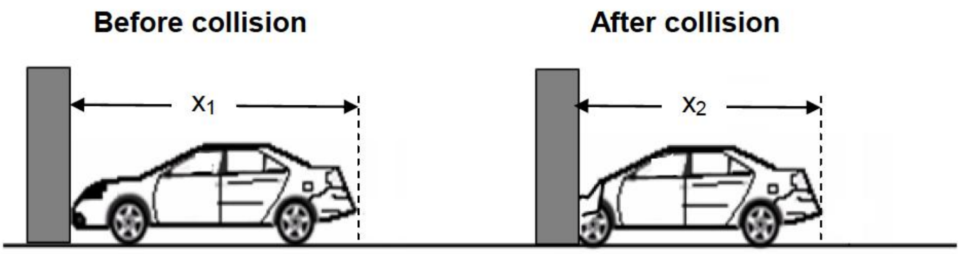
A. ΔE_K
B. ΔE_P
C. $-\Delta E_K$
D. $-\Delta E_P$

(2)

Question 2

(Adapted from DOE Nov 2012 Question 5)

In order to measure the net force involved during a collision, a car is allowed to collide head-on with a flat, rigid barrier. The resulting crumple distance is measured. The crumple distance is the length by which the car becomes shorter in coming to rest.



In one of the tests, a car of mass 1 200 kg strikes the barrier at a speed of 20 m·s^{-1}. The crumple distance, ($x_1 - x_2$), is measured as 1,02 m. (Ignore the effects of frictional forces during crumpling.)

a.) Draw a labelled free-body diagram showing ALL the forces acting on the car during the collision. (3)

b.) State the work-energy theorem in words. (2)

c.) Assume that the net force is constant during crumpling.

i.) USE THE WORK-ENERGY THEOREM to calculate the magnitude of the net force exerted on the car as it is brought to rest during crumpling. (4)

ii.) Calculate the time it takes the car to come to rest during crumpling. (4)

Links

- http://www.mindset.co.za/learn/xtra
- http://www.education.gov.za/Examinations/PastExamPapers/tabid/351/Default.aspx
- http://phet.colorado.edu/en/simulation/energy-skate-park